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Tensile strength and stress–strain behaviour of Devon silt under frozen fringe conditions

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ABSTRACT

Frost heave is attributed to the segregation of ice and ice lens formation as a soil freezes. Ice lens formation and hence frost heave starts with the cracking of the frozen fringe. In order for these cracks to initiate and open, the tensile strength of the soil has to be exceeded. Therefore, any evaluation of the ice lens initiation condition requires the determination of the tensile strength in the frozen fringe. Four point bending tests were carried out to determine the tensile strength of the frozen fringe and its stress-strain behaviour. Devon Silt samples frozen over a range of frozen fringe temperatures (0 to −1.5 °C) were tested at different deformation rates (0.08 mm/min to 8.0 mm/min). The frozen fringe of Devon silt has considerable tensile strength. The results show the dependency of the tensile strength on the temperature, the deformation rate, and the unfrozen water content. A unique strain rate dependency was determined for the frozen fringe. Further, it was observed that the stress-strain behaviour is influenced by the deformation rate and the subzero temperatures.

1. Introduction

The phenomenon of frost heave has been studied both experimentally and theoretically for decades (e.g. Taber, 1929; Beskow, 1935; Harlan, 1973; Konrad and Morgenstern, 1980; Gilpin, 1980). Experimental observations indicate that significant frost heave observed in field or laboratory is attributed to ice segregation and ice lens formation associated with water migration. Horizontal ice lenses and vertical ice veins are formed during freezing giving the reticulate ice lens structure observed in Fig. 1. Freezing-induced cracks are also formed during freezing as shown in Fig. 1. The presence of cracks during the freezing process has been observed by researchers such as Chamberlain and Gow (1979) and Arenson et al. (2008).

Three distinct zones exist in a freezing soil during frost heave: the passive frozen zone, the active unfrozen zone and the frozen fringe. These zones are shown in Fig. 1. The frozen fringe is the zone between the growing ice lens and the frost front where the warmest pore ice exists. Since the concept of frozen fringe was presented (Miller, 1972), it has been realized that the characteristics of the frozen fringe play a very important role in frost heave process. One-dimensional freezing tests by Xia (2006) showed the existence of freezing-induced cracks in the frozen fringe in tests carried out under various boundary conditions (Fig. 1). The freezing-induced cracks in the frozen fringe are very important features since they affect the rate of moisture migration through the frozen fringe during freezing and subsequently the formation of both vertical ice veins and horizontal ice lenses (Arenson et al., 2008, Azmatch et al., 2008). Hence, the damage caused by frost heave is affected by the presence and size of freezing-induced cracks.

To date, the exact nature of the formation of these cracks is not yet fully understood. They could be formed as a result of the moisture migration process, which may desiccate the unfrozen zone in the soil, in which case they may be referred to as desiccation cracks (Chamberlain and Gow, 1979), or they could be a result of the decrease in temperature, which results in thermal stress that may lead to cracking, in which case they may be referred to as thermal contraction cracks (Lachenbruch, 1962). However, there is not any published research that has established the exact nature of the formation of the cracks in the frozen fringe. Hence, we coined the term “freezing-induced cracks” to describe them, since the cracks are a result of the freezing process which can result in desiccation cracks or thermal cracks.

Independent on the nature of the cracks, i.e. desiccation cracks or thermal cracks, in order for the cracks to initiate, the tensile strength

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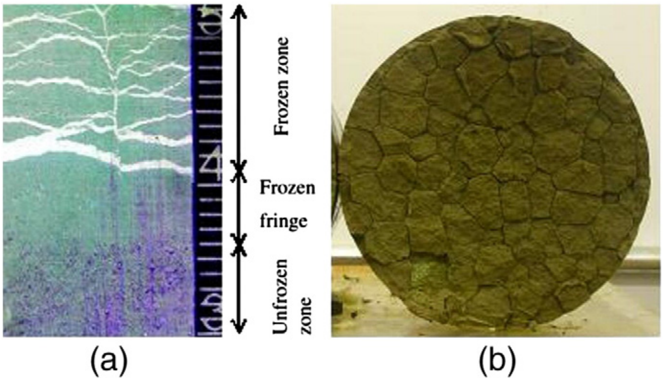

Fig. 1. Frost heave features: (a) Reticulate Ice lens structure and, (b) freezing-induced cracks in the frozen fringe (Xia, 2006).

of the soil has to be exceeded. Hence, investigation of the crack initiation process requires determination of the tensile strength of the soil over the temperature range of the frozen fringe.

Different properties of the frozen fringe were investigated by researchers such as Ping and Xiaozu (2000), Xiaozu and Ziawang (1997), Akagawa (1988), and Konrad and Morgenstern (1982). However, the tensile strength of the frozen fringe was not studied except by Akagawa and Nishisato (2009). It, however, is a very important parameter of the frozen fringe that needs farther investigation since it is thought to affect the cracking of the frozen fringe and hence the ice lens initiation condition and the hydraulic conductivity of the frozen fringe (Arenson et al., 2008, Azmatch et al., 2008). Hence, the tensile strength of the frozen fringe is investigated in this study.

Most of the research carried out on tensile strength of frozen soils has been at temperatures colder than found in the frozen fringe. It has been mainly at temperature colder than −5.0 °C (Haynes et al., 1975; Bragg and Andersland, 1980; and Zhu and Carbee (1985). Haynes (1978) conducted tensile strength tests on frozen silt at −0.1 and −1.7 °C. However, no effort has been made to specifically measure the tensile strength of the frozen fringe over the temperature range typically found in the fringe. An exception is the work by Akagawa and Nishisato (2009), who investigated the tensile strength of frozen soils over the temperature range of the frozen fringe. However, the tensile strength data they provided is widely scattered and does not show a well defined relationship with subzero temperature and the influence of strain rate and unfrozen water content was not investigated.

The tensile strength of frozen soils depends on temperature, loading rate and unfrozen water content. The effect of each has been investigated by different authors (Haynes et al., 1975; Haynes, 1978; Bragg and Andersland, 1980; Zhu and Carbee, 1985, 1987). Bragg and Andersland (1980) used split-cylinder tests to investigate the effect of strain rate on the tensile strength of frozen silica sand at a temperature of −6.0 °C. They found the tensile strength to be nearly independent of the deformation rate for values above 1.3 mm/min and at −6.0 °C. Haynes (1978) conducted direct tension tests to investigate the effect of temperature, loading rate and unfrozen water content on the tensile strength of Fairbanks silt. He conducted the tests over a range of temperature values (−0.1 °C to −57.0 °C) and over a range of strain rates ($1.6\times10^{-4}\ \mathrm{s}^{-1}$ to $2.9\ \mathrm{s}^{-1}$). He stated that the tensile strength doubled over the strain rate range and increased about one order of magnitude over the temperature range. Zhu and Carbee (1987) investigated the effect of temperature, strain rate and density on the tensile strength of Fairbanks silt by using direct tension test method. Their investigation was over a temperature range from −1.0 °C to −10.0 °C and over a deformation rate range of 5.9×10^{-4} mm/min to $5.\ 9\times10^{3}$ mm/min. The peak tensile strength of frozen silt was found to be very sensitive to strain rate. They concluded that for brittle failure, the peak tensile strength decreases slightly with increasing strain rate; and for ductile failure, it significantly decreases with decreasing strain rate. They determined that the peak tensile strength increases with decreasing temperature and that it increases more rapidly for temperatures colder than −5.0 °C. They also concluded that the initial tangent modulus is independent on strain rate. Christ and Kim (2009) used direct tensile test to investigate the effect of moisture content and temperature on the tensile strength of frozen silt over a temperature ranging from −2.0 °C to −20.0 °C. They observed a strong dependence of the stress-strain behaviour of frozen silt on the moisture content and temperature. Azmatch et al. (2010) investigated the tensile strength of frozen Devon silt using four-point bending test over a temperature range from −0.70 °C to −9.5 °C and using deformation rate from 0.8 mm/min to 8 mm/min. They observed an increase in tensile strength as the temperature decreased; and also that the tensile strength increased as the deformation rate increased.

Akagawa and Nishisato (2009) carried out tensile strength tests over the temperature range of the frozen fringe. They determined that there is a significant increase in tensile strength with a decrease in temperature. However, the tensile strength data they provided is widely scattered and does not show a well defined relationship with temperature and effect of strain rate was not investigated.

In this study, four-point bending test was used to investigate the tensile strength of the frozen fringe. The influence of temperature, strain rate and unfrozen water content on the tensile strength of the frozen fringe is investigated. Tests were also carried out at temperatures colder than the frozen fringe temperature range for the purpose of comparison.

1.1. Soil properties and sample preparation

The soil tested is Devon silt with a specific gravity of 2.65, a clay fraction of 25% and a silt fraction of 75%. It has a liquid limit of 32% and plastic limit of 20%. Slurry of the soil is prepared at a moisture content of 55% and then consolidated at 100 kPa in a consolidation cell. The moisture content at the end of consolidation is 27%. Soil samples of dimension 304.8 mm × 76.2 mm × 76.2 mm are then trimmed for the four-point bending test (FPBT). The dimensions in the test set-up are as shown in Fig. 2.

1.2. Soil sample freezing and freezing temperatures

The sample to be used for the FPBT is placed in a freezing cell. The temperature of the freezing cell is controlled by flowing cold fluid through brass coils (placed inside the freezing cell) from a cooling bath. The temperature of the freezing cell is monitored by two RTDs placed at two corners within the cell. The sample is let to freeze isotropically to the desired temperature once placed in the cell. The samples are first frozen at −4.0 °C for about 16 h and then the temperature is raised to the desired temperature and the sample is left to equilibrate at the desired test temperature for a minimum of

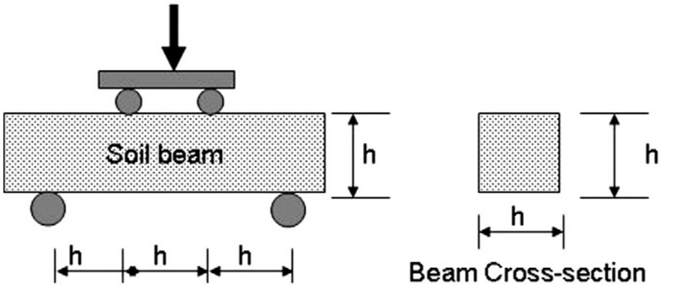

Fig. 2. Sample dimensions, h = 76.2 mm.

24 h. The samples are always inspected visually to see if there are any freezing-induced cracks that might influence the tensile strength of the sample. Cross sections along crack surface were also observed after testing to see if there are any special structures that may indicate presence of ice lenses at the center of the test specimen. In all the tests, no such features were observed.

1.3. Temperature range of the frozen fringe

The temperature in the frozen fringe decreases from the water freezing point at the frost front to the segregational temperature at the warm side of the ice lens. The segregation temperature is influenced by the overall temperature gradient in the sample (Xiaozu and Ziawang, 1997), the freezing rate (Konrad, 1989), the external load (Konrad and Morgenstern, 1982), the tensile strength of the soil (Akagawa et al., 2007) and the soil type (Konrad, 2005). Based on the segregation temperature values reported by Konrad (1989) for Devon silt, Ping and Xiaozu (2000) for Hebei Loam, and by Akagawa et al. (2007) for diluvial Dotan silt, temperatures ranging from 0 °C to −1.5 °C were used to carry out the tensile strength tests.

1.4. Tensile strength test using four-point bending test

After the sample reached the desired temperature, the flexural testing (FPBT) is carried out and digital images were taken at regular time interval during the test. A 15.1 megapixel digital camera (Canon EOS 50D) was used to take the images. The test set-up is as shown in Fig. 3. The digital images, together with the markers engraved on the sample, were used to determine the strains. The stresses were determined using beam flexure theory. It is assumed that the frozen soil is elastic; hence, an elastic analysis was carried out.

1.5. Unfrozen water content

Within a frozen soil, a certain amount of water remains unfrozen at subzero temperatures because of a decrease in the free energy of soil water due to surface forces associated with soil particles and the pore geometry among soil particles (Dash et al., 1995). The unfrozen water content was measured using time domain reflectometry (TDR). The TDR method measures the soil's dielectric property, which is converted to volumetric water content using the empirical equation described by Topp et al. (1980).

The TDR test was carried out on samples prepared in a similar manner to the samples used in the tensile strength testing. The temperature of the samples was measured by using RTDs placed within the samples. The results were then used to create the soil freezing characteristic curve (which is the unfrozen water content versus temperature). The unfrozen water content variation with temperature for Devon silt is shown in Fig. 4. The unfrozen water content curve indicates that there is a steep decrease in unfrozen water content in a temperature range from 0 °C to −1.0 °C. The change in unfrozen water content is small from −1.0 °C to −5.0 °C. Then the unfrozen water content remains almost constant at 6.5%.

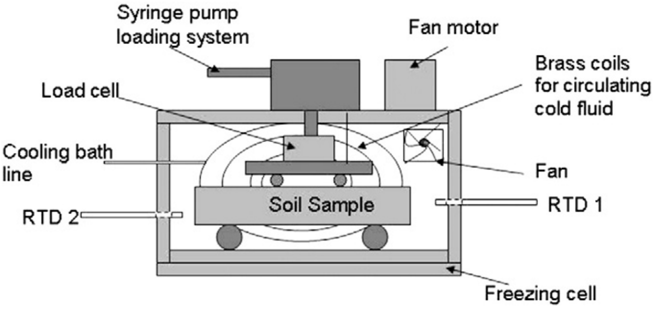

Fig. 3. Schematics of four-point bending test tension test set-up.

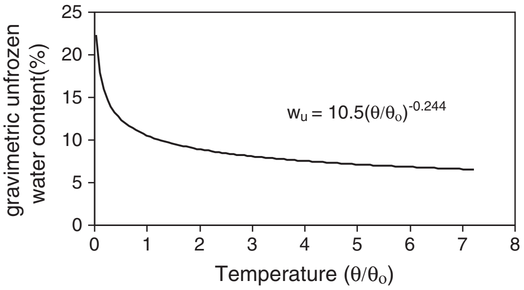

Fig. 4. Freezing characteristics (unfrozen water content) curve for Devon Silt (θ is the temperature at which the sample is frozen and θ_0 is a reference temperature taken as −1.0 °C).

The unfrozen water content changes drastically for temperature changes within the temperature range of the frozen-fringe. As the temperature changes from 0 °C to −1.5 °C, the unfrozen water content changes from 25% to 9.5%.

The dependence of unfrozen water content on temperature can be expressed as (Tice et al., 1976)

$$w_u = \alpha(\theta/\theta_o)^{\beta}$$

where θ is the negative temperature in °C; θ_o is a reference temperature taken as −1.0 °C; α and β are empirical parameters; and w_u is the gravimetric unfrozen moisture content expressed in percentage. For Devon silt consolidated at 100 kPa, the values of α and β are 10.5 and −0.244, respectively.

1.6. Experimental results and discussion

The frozen Devon silt showed a significant increase in tensile strength compared to its unfrozen state. The results from the tests carried out over the temperature range of the frozen fringe showed that the frozen fringe exhibits considerable tensile strength. Fig. 5 shows a sample loaded to failure. It is seen that the sample cracked in the middle span. All the samples tested cracked in this manner. The marks engraved on the sample are used to measure the strain development during the test.

1.7. Effect of subzero temperatures on tensile strength

To investigate the effect of subzero temperatures on tensile strength, tests were conducted at different temperatures ranging from −0.30 °C to −1.40 °C. These tests were carried out under a deformation rate of 0.8 mm/min. The temperature dependence of the tensile strength of the frozen fringe is shown in Fig. 6. The results show that the peak tensile strength of the frozen fringe is significantly influenced by the temperature; the tensile strength increases with decreasing in temperature.

The tensile strength of the unfrozen soil was also determined at a temperature of +2.25 °C. Devon silt in the unfrozen state has a peak

Fig. 5. Soil sample after loading.

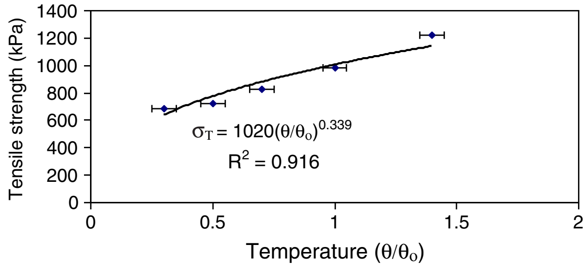

Fig. 6. Tensile strength as a function of temperature (θ is the temperature at which the sample is frozen and θ_0 is a reference temperature taken as −1.0 °C).

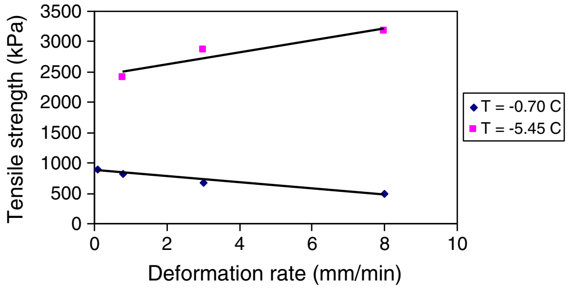

Fig. 8. Temperature dependence of the influence of deformation rate on the tensile strength of the frozen fringe.

tensile strength of 7.0 kPa under the test conditions in this study. An increase of two orders of magnitude (from 7.0 kPa to 686 kPa) is observed as the soil changed from an unfrozen state to a frozen state at a temperature of only −0.30 °C. This shows that the frozen fringe possesses considerable tensile strength.

Zhu and Carbee (1987) suggested a relationship for the peak tensile strength of frozen soils as a function of temperature as:

$$\sigma_T = A(\theta/\theta_o)^m$$

where θ is the negative temperature in ˚C, θ_o is a reference temperature taken as −1.0 ˚C, and A (in kPa) and m are empirical parameters.

Fig. 6 shows the variation of the peak tensile strength (σ_T) with temperature expressed as $\theta/\theta o$. It is determined that for the frozen fringe in Devon Silt under the conditions of investigation A = 1020 kPa and m = 0.339.

1.8. Effect of deformation rate on tensile strength of the frozen fringe

To investigate the effect of deformation rate on tensile strength of the frozen fringe, tests were conducted at different rates on samples frozen at −0.70 °C. The rates used were 0.08 mm/min, 0.8 mm/min, 3.0 mm/min and 8.0 mm/min. The results from these tests are presented in Table 1. The results are also plotted in Fig. 7.

The results show that the peak tensile strength is influenced by the deformation rate. For the tests carried out over the temperature range of the frozen fringe (at −0.70 °C), as the deformation rate increases, the tensile strength decreases (Fig. 7). This is contrary to existing knowledge on the influence of deformation rate on tensile strength of frozen soils (Haynes, 1978; Bragg and Andersland, 1980; Zhu and Carbee, 1987). However, the investigation on the influence of deformation rate by these researchers was carried out at colder temperatures; typically, -5.0 °C. To address this issue, tests were conducted tests at −5.45 °C for comparison to findings from the literature. The results from the tests carried out at −5.45 °C indicate that the tensile strength increases as the deformation rate increases (Fig. 8), which is in agreement with the results presented by other researchers. Fig. 8 shows that the effect of deformation rate on tensile strength is temperature dependent. The data by Akagawa and Nishisato (2009) supports the finding from this research. Akagawa and Nishisato (2009) conducted tensile strength tests at deformation rates of 0.34 mm/min and 2.31 mm/min on samples frozen at −0.16 °C and −0.15 °C, respectively. They found that the tensile strength under the lower deformation rate was about 30% higher than that under the higher deformation rate. However, they did not reach a similar conclusion for they had only two data points with a temperature difference of −0.01 °C. Hence, they suggested that the trend was due to small difference in freezing temperature. However, using the relationship established between temperature and tensile strength, it can be verified that a difference of −0.01 °C would not produce this much change in measured tensile strength.

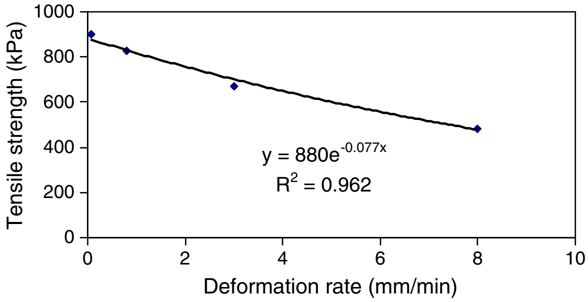

Fig. 7. Influence of deformation rate on the tensile strength of the frozen fringe. The deformation rate is the rate of displacement of compression test machine.

Hence, the behavior of frozen Devon silt under the frozen fringe temperature range is different from its behavior under colder temperatures. This behaviour is attributed to the change in unfrozen water content. There is a 67% increase in gravimetric water content as the temperature increases from −5.45 °C to −0.70 °C (from 7.0% to 11.7%). This change in water content could lead to a change in pore water pressure and hence a change in behaviour.

The variation of peak tensile strength with strain rate is shown in Fig. 9. Haynes (1978) expressed the tensile strength as a function of strain rate using:

$$\sigma_T = A\dot{\varepsilon}^b$$

where σ_T is the strength in kPa and $\dot{\varepsilon}$ is the strain rate in s^{-1}; A (in kPa) and b are constant for a given temperature. This equation, for Devon Silt, is presented in Fig. 9. The values of A and b are 297 kPa and −0.085, respectively.

1.9. Relationship between unfrozen water content and tensile strength

Using the temperature-tensile strength and temperature-unfrozen water content relationships, the relationship between unfrozen water

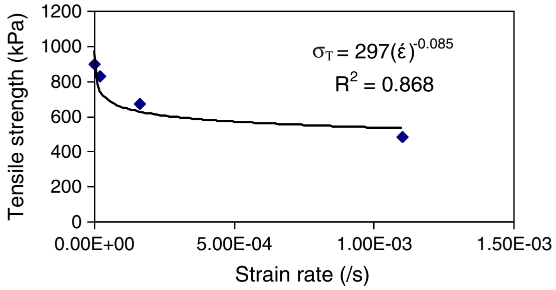

Fig. 9. Tensile strength as a function of strain rate at a temperature of −0.70 °C.

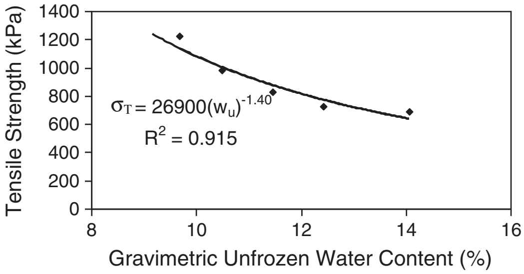

Fig. 10. Effect of unfrozen water content on tensile strength of the frozen fringe.

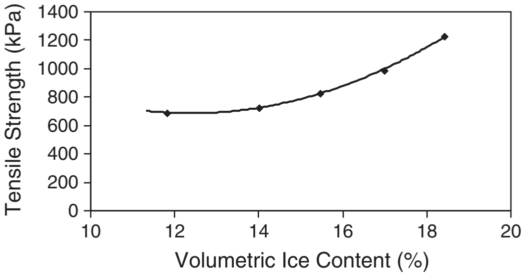

Fig. 11. Relationship between volumetric ice content and tensile strength.

content and tensile strength can be established. This relationship for Devon silt consolidated at 100 kPa is shown in Fig. 10. A change in unfrozen water content from 9.70% to 14.70% resulted in a change in tensile strength from 1220 kPa to 686 kPa.

1.10. Relationship between volumetric ice content and tensile strength

The relationship between volumetric ice content and tensile strength is shown in Fig. 11.

1.11. Stress–strain relationship and modulus of elasticity

The stress–strain diagrams for the tests conducted at different temperatures but at a deformation rate of 0.8 mm/min are shown in Figs. 12 and 13 shows the stress-strain relationships for the tests conducted at different deformation rates at a temperature of −0.70 °C. The digital images taken at different times during the test together with the linear marks engraved on the soil sample made the strain measurement possible. The change in length of the linear marks was measured using the ImageJ software.

1.12. Modulus of elasticity

The modulus of elasticity values were calculated from the initially linear portion of the stress-strain diagram. Fig. 14 shows the variation of modulus of elasticity with temperature. The modulus of elasticity increases significantly with a decrease in temperature. It is also influenced by deformation rate (Fig. 15).

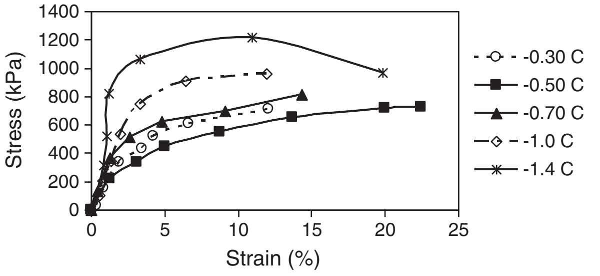

Fig. 12. Stress–strain plot for the tension tests at different temperatures for a deformation rate of 0.8 mm/min.

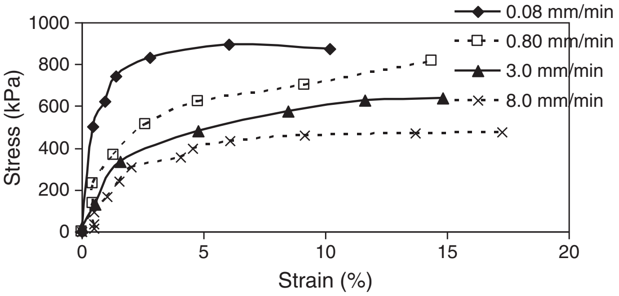

Fig. 13. Stress–strain plot for the tension tests at different deformation rates at a temperature of −0.70 °C.

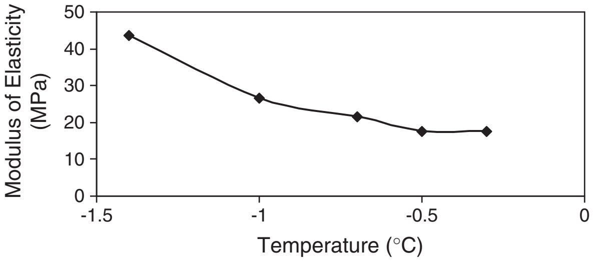

Fig. 14. Variation of modulus of elasticity with temperature for a deformation rate of 0.8 mm/min.

2. Conclusion

Four-point bending test was used to investigate the tensile strength and the stress-strain behaviour of the frozen fringe of Devon silt. The tests were conducted on samples prepared by consolidating slurry of Devon silt at 100 kPa. The influence of subzero temperatures, deformation rate/strain rate, and unfrozen water content on tensile strength of the frozen fringe of Devon silt was investigated.

The tests established that the frozen fringe possessed considerable tensile strength. Devon silt in the unfrozen state has a tensile strength of 7 kPa whereas it developed a tensile strength of 686 kPa at −0.30 °C.

To investigate the influence of subzero temperatures on tensile strength of the frozen fringe, tests were conducted at temperature values from −0.30 °C to −1.40 °C. The peak tensile strength increased as the temperature decreased. It changed from 686 kPa at −0.30 °C to 1220 kPa at −1.40 °C.

The influence of deformation rate on the tensile strength of the frozen fringe was investigate by carrying out tests at deformation

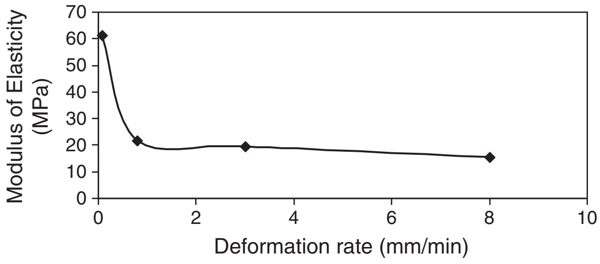

Fig. 15. Variation of modulus of elasticity with deformation rate for samples frozen at −0.70 °C.

rates of 0.08 mm/min, 0.80 mm/min, 3.0 mm/min and 8.0 mm/min on samples frozen at −0.70 °C. A unique behaviour of the frozen fringe was observed: the results showed that the peak tensile strength decreased as the deformation rate increased. It decreased from 900 kPa at 0.08 mm/min to 484 kPa at 8.0 mm/min.

The relationship established between gravimetric unfrozen water content and tensile strength showed that the tensile strength is influenced by the unfrozen water content. A small change in unfrozen water content produced a significant change in tensile strength. As the unfrozen water content decreased from 14.70% to 9.70%, the tensile strength increased by 78% (from 686 kPa to 1220 kPa).

The stress–strain plots showed that the modulus of elasticity is influenced by the temperature and the deformation rate: it increased from 17.60 MPa at −0.30 °C to 43.50 MPa at −1.4 °C; it decreased from 61.30 MPa at 0.08 mm/min to 15.70 MPa at 8.0 mm/min.

Additional tensile strength tests and one-dimensional freezing tests are currently being carried out to investigate the ice lens initiation condition and the formation of the freezing-induced cracks.

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